# Cointegration and Tests of Purchasing Parity

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*Most of us know Purchasing Power Parity as a sensible way of expressing per capita GNP; that is taking local price levels into account when making international comparison. Anthony Mac Guinness reminds us that the phrase also refers to a theory that the rates of price levels in two countries should determine their equilibrium exchange rates. This theory is then tested using the roles of cointegration analysis.*

*Editors Note: Appendices referenced in the text are available on the SER website. For reasons of space they have been omitted from this version.*

**Introduction**

This paper deals with the purchasing power parity approach to the analysis of exchange rates. Purchasing power parity (PPP) is commonly interpreted as the co-movement of the exchange rate and the relative price level of two countries. While this doctrine has failed to be empirically verified in the short run,[1] it still remains controversial as to whether this relationship holds in the long run, which is precisely the main focus of this paper. If PPP holds in the long run, inter-country commodity arbitrage ensures that deviations from a linear combination of spot exchange rates and domestic and foreign price levels should be stationary. Since a cointegrating system allows individual time series to be integrated of order one, but requires a linear combination of the time series to be stationary, then PPP is testable by using the theory of cointegrated processes.

This paper is organised as follows: The next section briefly presents some necessary preliminaries related to the relationship between exchange rates and the relative price levels between two countries postulated by PPP. Then, I will give a brief discussion relating to the statistical meaning of cointegration and the testing procedures thereof. My next section presents my empirical results while concluding remarks are contained in the final section.

[1] See Frankel (1981), Officer (1976) and Froot & Rogoff (1995) for an evaluation of the empirical investigations into the existence of PPP during the periods spanning both pre, post and during the Bretton-Woods era.

## CO-INTEGRATION AND TESTS OF PURCHASING POWER PARITY

### Purchasing Power Parity Theory

The basic notion behind PPP is that the ratio of domestic to foreign prices determines the *"fundamental"* or *"equilibrium"* exchange rate. The PPP hypothesis is stated as:

$$E = K\,(P^*/P)$$

Where E denotes the exchange rate measured as the number of units of domestic currency required to purchase a unit of foreign currency, and P* and P are the foreign and the domestic price levels respectively. Expressing all variables in logarithms:

$$E_t = a + b(P_t - P_t^*) + U_t \qquad (1)$$

An alternative representation expresses the domestic price level in terms of the exchange rate adjusted foreign price level.

$$P_t = c + d(E_t + P_t^*) + V_t \qquad (2)$$

If PPP holds exactly $b=d=1$, and changes in the domestic and foreign price levels are offset, except for stochastic shocks, by changes in the exchange rate. If PPP represents an equilibrium constraint, the exchange rate and the relative national price levels in (1) or the domestic price level and the exchange adjusted foreign price level (2) should form a cointegrating system. In essence two (or more) non-stationary time series are cointegrated if there exists a linear combination of the two, which is stationary. More precisely, consider two time series say $X_t$ and $Y_t$. Assume that both $X_t$ and $Y_t$ are non-stationary and need to be differenced once to induce stationarity. In general, most linear combinations of $X_t$ and $Y_t$, such as $X_t - aY_t = V_t$, are also non-stationary. If first differencing causes $X_t$ and $Y_t$ to be stationary, then $V_t$ will also be stationary after first differencing. However, there may exist a linear combination of $X_t$ and $Y_t$ that is stationary. For example, there may be a number $d$ such that $X_t - dY_t = U_t$, is stationary. In this special case, $X_t$ and $Y_t$ are said to be cointegrated of order (1,1) with a cointegrating vector of $d$, and the regression $X_t - dY_t = U_t$ is called the cointegrating or equilibrium regression[2].In the context of the application of cointegration theory to PPP, even though the individual time series have a tendency to drift away from a fixed value, the linear combination of these

[2] Johnston & DiNardo (1997)

two series tends towards zero[3]. Thus, a finding of cointegration is a necessary condition for PPP to qualify as a long run constraint, while rejection of cointegration implies that PPP has no tendency to hold, even in the long run.

**Testing for Cointegration**

Granger and Engle have proposed several tests for examining the hypothesis that two time series are cointegrated. In all seven tests, the null hypothesis is non-cointegration (against the alternative of cointegration). Therefore, a large test statistic rejects the null of non-cointegration that is, a large test statistic 'accepts' cointegration[4]. All of these tests involve estimating the so-called 'cointegration regression' of:

$$X_t = c + dY_t + u_t \qquad (1)$$

where $X_t$ and $Y_t$ are the series' being tested for cointegration. The test which I employ to investigate if the null of non-cointegration holds for equations (1) and (2) is a slightly modified Dickey–Fuller[5] type regression to test whether the estimated time series of the residuals from the cointegrating regression has a unit root: if here is a unit root, Xt and $Y_t$ are not cointegrated. The Dickey –Fuller, or DF, test uses the estimated residuals from equation (3) to estimate the regression:

$$\Delta\delta_t = -p\delta_{t-1} + \sigma_t \qquad (2)$$

where $\delta_t$ denotes the estimated residuals from equation (3).[6]The test involves the significance of the estimated *p* coefficient: if the estimated *p* is positive and significantly different from zero, the estimated residuals $\delta_t$ from the equilibrium equation are stationary so the hypothesis of cointegration is accepted.

[3] See Granger (1986) and Engle & Granger (1987).
[4] Note that the computed t-statistics do not follow the standard t distribution, nor is it asymptotically N(0,1), because stationarity was required in the derivation of the standard distributions. Dickey & Fuller (1979) computed the correct confidence intervals, which are wider then those computed using the student t distribution.
[5] Dickey & Fuller (1979).
[6] In the context of this paper the residual will be estimated from regressions (1) only.

## Empirical Results

*Data:*

Using monthly data obtained from *International Financial Statistics* database, I firstly test whether the variables of the cointegration regressions (1) are non-stationary and integrated of order one. Then on the bases of these results I will test for cointegration for both the consumer (CPI) and wholesale (WPI) price indices.[7]The exchange rate was the end of period market rate measured in terms of domestic currency per dollar.

*Model Estimated:*

The test of cointegration in this study were based on equation (1) repeated here for convenience:

$$E_t = a + b(P_t - P_t^*) + U_t$$

In the context of my data $P_t$ is the logarithm of the price index in Japan and the United Kingdom (UK); $P_t^*$ is the logarithm of the price index for the US price index, and $E_t$ is the logarithm of the exchange rate in units of domestic currency per US dollar. Both wholesale and consumer price index were tested. Firstly I test for (non) stationarity for each of the individual time series in the cointegration systems, which is equivalent to investigating the existence of a unit root in the time series. It is essential to establish that the individual time series are integrated of order one since a mixture of I(0) and I(1) variables are trivially cointegrated.

*Testing for Non-stationarity:*

The procedures which I employ to detect a unit root in the individual time series are: a visual investigation of the data to see if the autocorrelation functions for the

[7] The cointegration regressions are estimated for both price indexes to investigate if the use of the (more traded good weighted) WPI provides more supportive results in favour of PPP then the (more non-traded good weighted) CPI which would not be expected to be supportive of PPP due to the presence of information and transport cost, trade impediments, price discrimination, differences in the weights used in price indices, productivity differentials etc. While the WPI is also exposed to these biases it is plausible to believe that the basket of traded goods would be more in line with the original notion of equality in exchange rate adjusted price levels proposed by the founders of the PPP relationship.

individual time series exhibit non-stationary trends and secondly I apply the Augmented Dickey-Fuller (ADF) test to the time series. The ADF test accounts for temporally dependent or serially correlated distributed error terms by including lagged innovation sequences in the fitted regression. The form of this regression equation is as follows:

$$\Delta Y_t = \beta_1 + \beta 2\ Y_{t-1} + \alpha_1 \sum^{m}_{i=1} \Delta Y_{t-i} + \delta_t \qquad (1)$$

The null hypothesis in this test is that $\delta=0$, that is, there is a unit root.
The results from applying the ADF statistics to the level of the logarithms of spot exchange rates and both wholesale and consumer price index differentials between the domestic and the foreign countries are presented in *Tables 1 and 2*. The AIC and the SBC (two information criteria) indicate the order of lagged innovation terms, which should be chosen for each ADF test. *Table 1* provides evidence that the UK and the Japanese spot exchange rates are non-stationary. All computed t-statistics for the Augmented Dickey-Fuller tests with and without a trend (see footnote 9) are less in absolute value than the critical value and evidently are integrated of order one. Thus implying non-stationarity and the existence of a unit root. This is also evident from the autocorrelation functions, all of which exhibit a non-stationary trend supportive of the results presented in *Table1*[8].

*Table 1*[9] *Unit Root Tests for the Spot Exchange Rate* (Logarithm)*

| *Country* | *ADF* | *Critical value*** |
|---|---|---|
| UK | -2.2570 | -2.8712 |
| | -2.3319 | -3.4260 |
| JAPAN | -1.2099 | -2.8712 |
| | -1.8390 | -3.4260 |

*The spot exchange rate is domestic currency per dollar.
** This is the 95% critical value for the augmented Dickey-Fuller.

[8] Note both the Box Pierce Q-stat and the Ljung-Box (LB) statistic are both highly significant in all samples.
[9] The first row for each country gives the computed ADF statistic and associated critical value for the spot exchange rate as defined above with an intercept while the second row gives the same results however including a linear trend in the ADF regression. Both values are presented here for completeness, however the ADF without trend is more suitable as the graphical representation of these two spot values does not suggest a trend is likely.

*Table 2* gives a contrasting picture for the Japan and the UK experience. Results for both the consumer and the wholesale relative price levels exhibit non-stationarity in the Japanese price series data thus confirming the evidence presented by the autocorrelation function for both these price series. However the UK price series data is found to be non-stationary on the evidence provided by the Augmented Dickey-Fuller test.[10] This implies that while the spot exchange rate series for the UK is integrated of order one, both the consumer and the wholesale relative national price levels are I(0). This implies that one cannot take a linear combination of the spot exchange rate in logarithm form. Also the difference between the log of the price series' for the UK and the US form a cointegrating system as both are not of the same order of integration. Thus on the evidence presented in this paper PPP does not hold (in the long run) for the UK and the US on the *a priori* assumption that cointegration is a necessary condition for PPP to qualify as a long run constraint, as outlined above.

*Table 2 Unit Root Tests for the Consumer and the Wholesale Price Indices (Logarithm)*

| *Country* | *Price Index* | *ADF* | *Critical Value* |
|---|---|---|---|
| UK | CPI | -5.2826 | -2.8712 |
| | | -4.0270 | -3.4260 |
| | WPI | -5.0663 | -2.8712 |
| | | -3.5322 | -3.4261 |
| Japan | CPI | .59408 | -2.8712 |
| | | -3.0097 | -3.4260 |
| | WPI | -1.6023 | -2.8712 |
| | | -2.9391 | -3.4260 |

*Cointegration Testing:*

The tests for cointegration in this study were based on equation(1), repeated here for convenience:

$$E_t = a + b(P_t - P_t^*) + U_t$$

Cointegration requires that the estimated residuals from equation one are stationary. The hypothesis of a unit root in the residual series was tested with both the DF and the ADF statistics, with approximate critical values from MacKinnon (1991).

[10] This conflicts with the visual evidence in support of nonstationarity in the price series' for the UK provided on investigation of the autocorrelation function.

*Table 3* (see note with table explaining structure) reports the cointegrating regression estimates and test statistics for equation (1), testing whether the spot exchange rate and the relative national price levels for Japan and the US are cointegrated, Test were not preformed for long run PPP between the UK and the US as the relative national price levels were found to be I(0) for both the CPI and the WPI thus implying that the spot exchange rate and the relative national price levels are trivially cointegrated.

The evidence presented in *Table 1* does not reject the null hypothesis of a unit root in the residuals series for either the WPI or the CPI, thus rejecting my *a priori* proposed hypothesis that the use of the more traded good weighted WPI would provide more support for PPP as tested using cointegration theory then the CPI (See footnote 7). Thus on the bases of these results I cannot accept that deviations from PPP in the long run converge to some equilibrium path and thus must accept that shocks to the error term persist in the long run. These results are consistent with the autocorrelation functions for the estimated residuals from equation (1).

*Table 3. Cointegration Regression**

| Depend-ent variable | Intercept | Slope | $R^2$ | T | ADF | DF | C.V. | DW |
|---|---|---|---|---|---|---|---|---|
| | (Standard error) | (Standard error) | | | | | | |
| Japan | | | | | | | | |
| E (CPI) | 1.9101 | 4.777 | .78897 | 318 | -2.1454 | | -3.3574 | .04562 |
| | (.05557) | (.014876) | | | | | | |
| E(WPI) | 4.7039 | 1.4789 | .89588 | 318 | | -2.2855 | -3.3574 | .0767 |
| | (.011159) | (.028361) | | | | | | |

*Reported here are the cointegration regressions defined by equation (1) with the nominal exchange rate as the dependent variable and the price index used to construct the explanatory variable indicated in parenthesis. All variables are in logarithm form. The column headed by T gives the number of observations, DF and ADF are the computed t- statistics for the non-augmented and the augmented Dickey-Fuller tests of the unit root for the residual series, and DW is the Durbin-Watson statistic from the cointegration regression. If no lagged differences are

significant, no ADF is reported. C.V. stands for the computed 95% Dickey-Fuller critical values.

Another method of testing whether PPP holds in the long run is by testing for unit roots in the logarithm of the real exchange rate series:

$$R_t = E_t - P_t + P_t^* \quad (1)$$

Stationarity of the real exchange rate implies that departures from PPP are self - correcting.[11] This method may give weaker support for purchasing power parity due to the strong restrictions imposed[12] relative to those in the cointegration regression. Departures from a coefficient of unity in equation (1) may be consistent with cointegration, but only if this coefficient is precisely one will the real exchange rate tend to a constant value in the long run. The results presented in *Table 4* are from applying ADF tests to the real exchange rate in order to test the absolute version of PPP. I cannot reject the null hypothesis that the real exchange rate has a unit root thus implying that the real exchange rate, for both countries considered, is a random walk. In other words, deviations from PPP have no tendency to converge to a long run equilibrium path. More over, linear regressions involving the domestic and foreign relative national price levels and the spot exchange rates can only be interpretated as spurious. This conclusion is derived from the suggested rule of thumb by Granger and Newbold (1974), i.e. when the $R^2$ is greater than the DW statistic it is highly likely that the regression results are spurious.

*Table 4. Unit Root Tests for the Real Exchange Rate**

| *Country* | *ADF* | *Critical value*** |
|---|---|---|
| UK | -2.4825 | -2.8712 |
| | -2.5091 | -3.4260 |
| JAPAN | -1.8568 | -2.8712 |
| | -1.9574 | -3.4260 |

*The ADF statistic for the real exchange rate based on the CPI is presented in the first row for each country while for the real exchange rate based on the WPI is presented in the second row for each country.
** This is the 95% critical value for the augmented Dickey-Fuller.

[11]Froot & Rogoff (1995)
[12]ibid

**Relative PPP**

While this paper concentrates on absolute PPP in the long run, economically there is no reason why this relationship should hold unless both countries have identical consumption baskets. In order to allow for a constant price differential between baskets, I assess briefly the concept of relative based PPP, denoted as:

$$\Delta E_t = a + b\Delta(P_t - P_t^*) + Z_t \quad (1)$$

which requires that the changes in relative price levels are offset by changes in the exchange rate.[13]

*Table 5*[14] reports the estimates and the test statistics from this equation for both countries. It is evident that the null hypothesis of a unit root in the residuals estimated from equation 7 is rejected for both countries considered. The conclusion of stationarity of the residuals is also supported by the autocorrelation functions of the estimated residuals.

One interesting feature of non-stationarity in the residuals, (as mentioned in footnote 3), is that the t-statistics are not reliable and many other diagnostic and inference tests are flawed also. One implication of this concerns test regarding the structural stability of the model, namely the CUSUM and the CUSUM of squares technique.[15] While for relative PPP the residuals are concluded to be stationary and the CUSUM and the CUSUM of squares provide evidence of structural stability, however in the case of absolute PPP the CUSUM and the CUSUM of squares breach the standard error bands thus indicating evidence of a structural break in the data. However, these findings are of dubious value as the standard error bands in the case of absolute PPP are not correctly specified due to the existence of a unit root in the residuals.

[13]Froot & Rogoff (1995)
[14]Table 5's structure is consistent with that of Table 4.
[15] Both the CUSUM and the CUSUM of squares plots, test the regression equation's structural stability. Based on the scaled recursive residuals, the plot will not cross the 5% critical bands if the equation is correctly specified.

*Table 5. Cointegration Regression**

| *Dependent variable* | *Intercept* | *Slope* | $R^2$ | *T* | *DF* | *DW* | *C.V.* |
|---|---|---|---|---|---|---|---|
| | (Standard error) | (Standard error) | | | | | |
| UK | | | | | | | |
| ΔE (CPI) | .0019262 | -.31018 | .0042212 | 317 | -15.7990 | 1.8017 | -3.3574 |
| | (.0018368) | .26843 | | | | | |
| ΔE(WPI) | .0013448 | -.044383 | .3642E-3 | 316 | -15.8556 | 1.8109 | -3.3575 |
| | (.0017855) | (.13122) | | | | | |
| Japan | | | | | | | |
| ΔE (CPI) | -.0029326 | -.052911 | .1098E-3 | 317 | -16.1396 | 1.8247 | -3.3574 |
| | (.0019496) | (.28448) | | | | | |
| ΔE(WPI) | -.8957E-3 | .97119 | .061759 | 317 | -17.3035 | 1.9911 | -3.3574 |
| | .0019135 | .21328 | | | | | |

*Reported here are the cointegration regressions defined by equation (1) with the nominal exchange rate as the dependent variable and the price index used to construct the explanatory variable indicated in parenthesis. All variables are in logarithm form. The column headed by T gives the number of observations, DF and ADF are the computed t- statistics for the non-augmented and the augmented Dickey-Fuller tests of the unit root for the residual series, and DW is the Durbin-Watson statistic from the cointegration regression. If no lagged differences are significant, no ADF is reported. C.V. stands for the computed Dickey-Fuller 95% critical value.

## Conclusion

This paper tests whether the absolute version of PPP holds in the long run through the theory of cointegration. If PPP holds, then the residuals from the cointegrating process, and the real exchange rate will be stationary. Using ADF statistics I have been unable to reject the null hypothesis of a unit root in both instances regardless of whether these expressions are in terms of the wholesale or consumer price indices. In the case of the UK, I found that the relative national price levels with the US were stationary thus implying that the spot exchange rate and the relative price series were trivially cointegrated. These results imply that PPP does not hold as a long run constraint and that deviations from PPP do not converge to some long run path as postulated by the absolute version of the purchasing power parity relationship. Results related to the relative version of PPP, while supportive of the theory in the sense that the residuals from this cointegrating system are stationary, are unable to justify that changes in the relative price level between two countries will be offset in the long run by changes in the exchange rate, as regressions of this form lose any long run relationship.[16]

While the tests employed in this paper have been substantiated within the body of empirical testing of PPP and its deviants, certain concerns surround these approaches. With respect to test employing the random walk approach to the real exchange rate there exist concerns regarding insufficient power to reject the null. Froot and Rogoff (1995) have shown that to be able to reject the null of a unit root for a single currency at a 5% critical interval using a large sample Dickey-Fuller critical t value of 2.89 would imply a time series of 864 months or 72 years. Two approaches to this problem are: employ a longer time horizon including both pre- and post-Bretton Woods periods; or to employ the technique of panel data, i.e. look at a cross section of currencies simultaneously. A second concern is with regards cointegration theory. This essentially is related to the large variations of the cointegration vector across varies studies based on modern floating rate data, which makes it difficult to interpret the results from cointegration tests. Froot and Rogoff attribute these wide-ranging deviations in coefficient estimates to small sample bias. They suggest that regressions with $R^2 < 0.95$ are likely to lead to substantial bias and

[16] Gujarati (1985)

highlight the fact that small sample bias is especially prevalent in exchange rate regressions over floating rate data.

Frenkel (1981) argued that much of the controversy over the use of PPP doctrine results from the fact that PPP specifies a final, equilibrium relationship rather then a precise theory of exchange rate determination. If cointegration is interpreted as evidence of long run PPP then the results provided in this paper suggest that prices and exchange rates diverge even in the long run. While it should be noted that prior empirical studies concerning PPP between industrialized countries have been unable to support this relationship, studies concerning highly inflated economies (relative to the US) have been more supportive of the relationship. This contrast can be attributed to the dominance of monetary relative to real factors in these economies thus implying that while PPP provides sufficient explanatory power between prices and exchange rate movements in highly inflated economies, it understates the role of real disturbances in industrialized countries and the supply side determinants emphasized by the popularised Ballassa-Samuelson model which attribute to its subsequent insufficiency in explaining the co-movement of prices and exchange rates in these economies.

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